SUM RULES AND 2-QUARK FLUX-TUBE STRUCTURE

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Sum rules – relating the static quark potential $V(R)$ to the spatial distribution of the action and energy in the colour fields of flux-tubes – are applied in three ways:
1) To extract generalised β-functions:
2) As a consistency check for the use of excited gluon flux-tubes and as an estimate of the quark self-energies:
3) To extract approximate sum rules using a simplified form of $V(R)$.
Also the flux-tube profiles are compared with hadronic string and flux-tube models.

1 Introduction

In ref. [1] sum rules were derived to relate the sum over all spatial positions of colour fields ($\mathcal{E},\ \mathcal{B}$) to the static quark potential $V(R)$ and its derivative:

$$\frac{-1}{b}\left(V + R\frac{\partial V}{\partial R}\right) + S_0 = -\sum(\mathcal{E}_L + 2\mathcal{E}_T + 2\mathcal{B}_T + \mathcal{B}_L) \tag{1}$$

$$\frac{1}{4\beta f}\left(V + R\frac{\partial V}{\partial R}\right) + E_0 = \sum(-\mathcal{E}_L + \mathcal{B}_L) \tag{2}$$

$$\frac{1}{4\beta f}\left(V - R\frac{\partial V}{\partial R}\right) + E_0 = \sum(-\mathcal{E}_T + \mathcal{B}_T)\ . \tag{3}$$

Here $L(T)$ refers to longitudinal(transverse) with respect to the interquark separation axis. The $\mathcal{E}(\mathcal{B})$ have the interpretation of gauge invariant averages of the fluctuation of squared strengths of the colour electric(magnetic)fields. The parameters b and f are related to the generalised β-functions

$$\frac{\partial\beta_{ij}}{\partial \ln a_k} = S \text{ if } k = i \text{ or } j \quad \text{and} \quad \frac{\partial\beta_{ij}}{\partial \ln a_k} = U \text{ if } k \neq i \text{ or } j \tag{4}$$

by the expressions

$$b = 2(S{+}U) = -0.3715(1{+}0.49\alpha{+}\ldots) \quad , \quad 2\beta f = U{-}S = 2\beta(1{-}1.13\alpha{+}\ldots)\ , \tag{5}$$

where the perturbative series for these quantities in terms of the bare lattice coupling $\alpha = g^2/4\pi = 1/\pi\beta$ for $SU(2)$ colour fields are also given [2]. The S_0 and E_0 are the self energy terms. The general strategy for utilizing these sum rules is to insert known values or forms of $V(R)$ on the LHS and to measure on a lattice the $\mathcal{E}$, $\mathcal{B}$ on the RHS. In this talk three such applications are discussed. In addition to these integrated colour field quantities, the individual field profiles are also studied.

2 Three applications of the above sum rules

The $V(R)$, $\mathcal{E}$ and $\mathcal{B}$ are all calculated in quenched SU(2) on the same $16^3 \times 32$ lattice with $\beta = 2.4$ using basis states with different degrees of fuzzing. The latter enables the excited gluon states with cubic symmetry E_u and A'_{1g} to be studied.

1.1 $V(R)$ directly from the lattice to give Beta-functions

With $V(R)$ calculated on the same lattices as the $\mathcal{E}, \mathcal{B}$, combinations of the above sum rules are made at two values of R to eliminate the $\frac{\partial V}{\partial R}$ and also the self-energy terms giving:

$$f = -\frac{[V(R_1) - V(R_2)]}{2\beta\,[(A+B)_{R_1} - (A+B)_{R_2}]} \tag{6}$$

$$b = \frac{2[V(R_1) - V(R_2)]\left[1 + \frac{A_{R_1} - A_{R_2}}{B_{R_1} - B_{R_2}}\right]^{-1}}{C_{R_1} - C_{R_2}}\ , \tag{7}$$

where

$$A_R = \sum(\mathcal{E}_T - \mathcal{B}_T)_R \quad , \quad B_R = \sum(\mathcal{E}_L - \mathcal{B}_L)_R$$
$$C_R = \sum(\mathcal{E}_L + 2\mathcal{E}_T + 2\mathcal{B}_T + \mathcal{B}_L)_R.$$

This results in best estimates of b = –0.35(2) and $f = 0.61(3)$. Therefore, b is seen to be far from the perturbation estimate of –0.42 with $\alpha_{effective} = 0.26$. A more detailed account can be found in ref. [3]

1.2 A parametrization of the lattice potential $V(R)$

Armed with estimates for b and f, Eqs. 1-3 can be studied separately using the following interpolations of the lattice $V(R)$ for the ground (A_{1g}) and first excited (E_u) state:

$$V(R)_{A1g} = 0.562 + 0.0696R - 0.255/R - 0.045/R^2 \tag{8}$$

$$V(R)_{Eu} - V(R)_{A1g} = \pi/R - 4.24/R^2 + 3.983/R^4. \quad (9)$$

For the ground state case it is found that all three sum rules are well satisfied with S_0=0 and E_0=0.1. However, the comparison is not so good for the E_u state – indicating that this state is still somewhat contaminated for the values of Euclidean time $T \leq 4$, for which a signal could be measured. More details can be found in ref. [4]

1.3 A simple parametrization of $V(R)$

If the form $V(R) = c + b_s R + e/R$ is used in the sum rules, then they reduce to

$$c + 2b_s R = \sum 2(S+U)[\mathcal{E}_L + 2\mathcal{E}_P + 2\mathcal{B}_P + \mathcal{B}_L] \quad (10)$$

$$c + 2b_s R = \sum 2(S-U)[\mathcal{E}_L - \mathcal{B}_L] \text{ and } c + \frac{2e}{R} = \sum 2(S-U)[\mathcal{E}_P - \mathcal{B}_P], \quad (11)$$

where $e \approx -0.25(\pi)$ for the ground(E_u) state. Since Eqs. 10 and 11a are independent of e, they reduce further to

$$\sum_{x,y,z}[S^{\rm Ex}] = \sum_{x,y,z}[S^{\rm GS}] \quad \text{and} \quad \sum_{x,y,z}[E_L^{\rm Ex}] = \sum_{x,y,z}[E_L^{\rm GS}] \quad (12)$$

and the third becomes

$$\frac{\Delta e}{R} \approx \beta f \sum_{x,y,z}[E^{\rm Ex} - E^{\rm GS}], \quad (13)$$

where $S^i(E^i)$ refers to the action(energy) combinations of $\mathcal{E}(\mathcal{B})$ in Eqs. 10, 11. For sufficiently large R, the replacement $\sum_{x,y,z} \to \frac{R}{a}\sum_{x,y}$ should be a reasonable approximation. In this case the 3-d sum rules in Eqs. 12,13 reduce to 2-d sum rules involving less Monte Carlo data and are useful when discussing flux-tube profiles in the next section. More details can be found in ref. [4]

3 Flux-tube profiles

So far only the sum rules for combinations of $\mathcal{E}$, $\mathcal{B}$ have been discussed. In this section the non-integrated combinations of $\mathcal{E}, \mathcal{B}$ are compared with two models. In Fig. 1a comparison is made for energy profiles at the centre of the line connecting the two quarks. There MC refers to the Monte Carlo result at $R = 6$, BBZ to the dual superconducting model of ref. [5] and IP to the string motivated model of ref [6]. In the latter, the string energy was tuned to fit the MC result on the central axis. The disagreement with the other two curves is

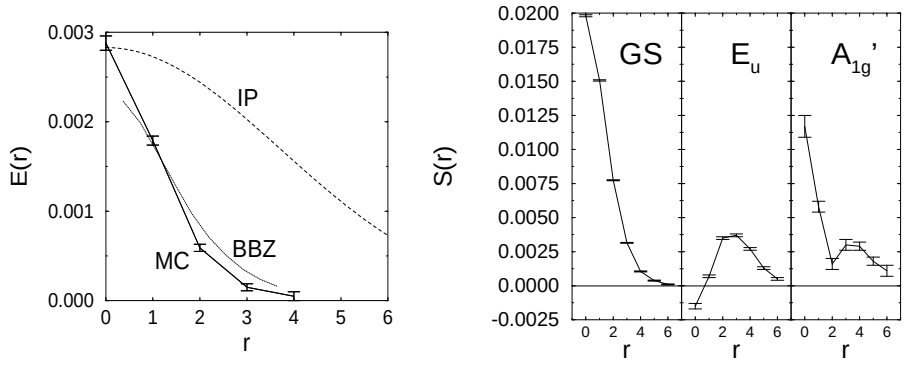

Figure 1: (a) The Energy profile $E(r)$ for $R = 6$: (b) The Action profiles $S(r)$ for the states A_{1g}, E_u and A'_{1g}, calculated at $R = 6$. Both $E(r)$ and $S(r)$ are dimensionless and r – the distance from the central axis – is in lattice units of $a = 0.12$fm.

not surprising, since the IP model includes a sizeable zero-point energy in its description of the gluon fields. Therefore, only *differences* between the profiles of the separate states should be compared. The similarity between the MC and BBZ results can then be interpreted as the two having similar self-energies . In Figs. 1b the action profiles for the ground and E_u, A'_{1g} states are shown. Here, it is of interest to see that the A'_{1g} case has a dip at $r \approx 2a$ and is reminiscent of the node expected in excited s-wave states. The total action and energy profiles for the ground state are in agreement with refs.[7] [8].

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References

1. C. Michael, *Phys.Rev.* **D53**, 4102 (1996).
2. F. Karsch, *Nucl. Phys.* **B205**, 285 (1982) .
3. C. Michael, A.M. Green and P.S. Spencer, hep-lat/9606002 – to be published in *Phys.Lett.B*
4. A.M. Green, C. Michael and P.S. Spencer, in preparation
5. M. Baker, these Proceedings; M. Baker, J.S. Ball and F. Zachariasen, *Int.Jour.Mod.Phys.* **A11**, 343 (1996)
6. N. Isgur and J.E. Paton, *Phys. Rev.* **A31**, 2910 (1985).
7. G. Bali, K. Schilling and C. Schlichter, *Phys. Rev.* **D51**, 5165 (1995).
8. R.W. Haymaker, V. Singh and Y. Peng, *Phys.Rev.* **D53**, 389 (1996).